FET Statistical Modeling Using Parameter Orthogonalization

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Abstract—**A new method for representing the statistical variation of FET equivalent circuit parameters (ECP's) is presented. This method utilizes a statistical technique known as principal components and provides an efficient method for statistically representing the means, standard deviations, and correlations of the FET ECP's. The technique can easily be implemented into commercial CAD simulators resulting in FET variation simulations that are more accurate than existing methods. Appropriate statistical tests for determination of equivalence between simulated and measured FET parameter distributions is also discussed. Both the modeling methodology and statistical testing were demonstrated using both scattering and noise parameters for 300 μm low-noise GaAs FET's.**

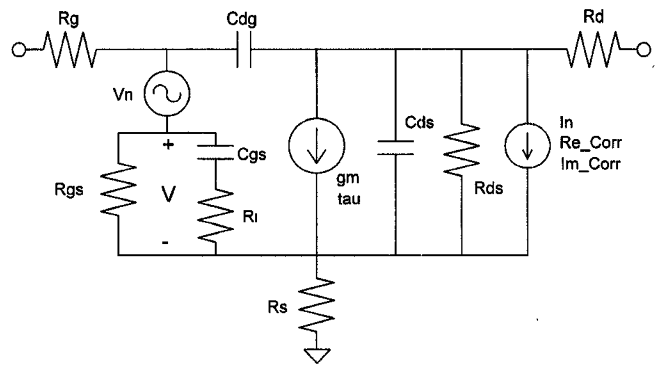

Fig. 1. Small signal model including intrinsic, extrinsic resistances, and noise elements.

I. Introduction

STATISTICAL CIRCUIT modeling has shown increasing popularity with microwave circuit designers during the last few years [1]. This is due to the incorporation of statistical yield analysis and optimization into commercial computer-aided design (CAD) programs. Statistical modeling allows the microwave engineer to evaluate circuits on the basis of their producibility as well as good electrical performance. This results in more reliable, higher yielding products which are more commercially competitive.

The foundation for most CAD yield analysis and optimization tools is the Monte Carlo method [1]. It is well known that all circuit parameters vary randomly around their nominal, or "designed," values due to fluctuations inherent to the production of the circuit. These fluctuations are due to each component's intrinsic tolerance which is governed by technological and cost considerations. For example, GaAs microstrip may be designed to be 75 μm wide but may vary ± 2.5 μm due to gold plating limitations. The random fluctuations in the circuit components causes a corresponding variation in the circuit response. Commercial microwave CAD packages use the Monte Carlo technique to model these processing fluctuations as statistically independent, random variables in order to predict how the circuit will respond. However, many fluctuations within a circuit cannot be expressed in the form of independent random variables. A common and very influential example of correlated variables in microwave circuit modeling are the small signal FET model parameters [2].

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Fig. 1 illustrates a conventional small signal FET model for noise and S-parameter characteristics over frequency [3]. The model gives reasonable results for small signal conditions by including the intrinsic, some extrinsic, and noise parameters. The model's main strength is its compactness and ease of use in the CAD modeling environment. The potential for FET parameter scalability is also an advantage which cannot be ignored [4]. However, the small signal FET ECP's are described by highly correlated multivariate distributions [2] and therefore cannot be easily implemented in existing commercial CAD software for Monte Carlo simulation. Some designers have tried to model the FET parameters as independent random variables with mixed success [2], [5], and [6]. Due to the physical correlations existing between FET parameters this modeling scheme, referred to as the plus-minus sigma ($\pm\sigma$) model, can often result in physically impossible FET parameter combinations during a Monte Carlo simulation. This situation is undesirable if truly accurate CAD yield predictions are required.

In order to remedy the shortcomings of the correlated FET parameters the Truth Model has been suggested in [5] and [7] and successfully implemented into commercial CAD packages [8]. This method is simple and inherently creates the orthogonality of random variables that the Monte Carlo method requires. In fact, the Truth Model can be thought of as making the entire FET one random variable picked from an S-parameter database during the Monte Carlo simulation. However, large S-parameter databases are needed to cover all of the frequency ranges and bias conditions necessary for accurate statistical modeling. The Truth Model is not compact and cannot be scaled to different FET sizes as can the small signal FET model. Also, the randomness of Monte

Carlo samples is severely limited by the size of the S-parameter database. Campbell *et al.* have suggested database interpolation in order to reduce the impact of this limitation. However, this interpolation results in an even larger database for random FET selections [9] and does not solve the inherent problem. Finally, S-parameter database access time must be considered for a large number of Monte Carlo simulations. In summary, although the Truth Model is accurate, it limits the economic feasibility of statistical design due to complexity, database requirements, and computational inefficiencies.

Purviance *et al.* suggested statistically characterizing FET's through the use of a principal component analysis of the S-parameters database [10]. This solution has many of the disadvantages of the Truth Model the most important of which is the large database needed for the statistical modeling at different frequencies, FET sizes, and biases. However, it will be shown herein that the principal component technique can also be applied to the small-signal FET parameters to obtain an accurate and compact statistical model for circuit simulation of both noise and S-parameters.

This paper proposes application of the principal components statistical technique to a small signal FET equivalent circuit parameter database. We will show that the correlated parameters can be easily expressed in terms of uncorrelated random variables suitable for Monte Carlo analysis. The FET parameter's means, standard deviations, and correlations will be shown to be preserved during a Monte Carlo simulation. An example of the methodology will be presented for a population of 300 μm low-noise GaAs FET's by incorporation of the principal component technique into a currently available CAD microwave simulator. Statistical tests presented in this paper will be used to verify the improvement of simulated FET noise and S-parameters over the traditional $\pm\sigma$ model.

II. Principal Components

Principal component analysis is a well-known statistical technique by which a sample data set of n correlated variables are linearly transformed into a new data set of n uncorrelated, or orthogonal, variables called principal components [11]. Statistically, correlation is defined as the linear relationship between two or more variables. Essentially, the principal component technique rotates the variable axes in order to obtain data with no linear relationships. Fig. 2(a) shows a set of data points which obviously have a strong positive linear relationship which respect to the X and Y coordinate system. Principal components effectively rotates the axes to produce a new coordinate system described by $F1$ and $F2$. The data in Fig. 2(b) is uncorrelated when referenced to this new coordinate system. The same concept can be applied to an n-dimensional coordinate system of a sample data set resulting in a new n-dimensional data set referenced to the orthogonal principal component axes. Mathematically, this rotation is achieved by determining the eigenvalues of the $n \times n$ correlation matrix of a sample data set. Equation (1) shows the vector $\mathbf{E}$ containing the FET parameter variables from Fig. 1, the vector $\mathbf{F}$ which contains the orthogonal principal components, Λ the diagonal eigenvalue matrix, and $\mathbf{U}$ a matrix

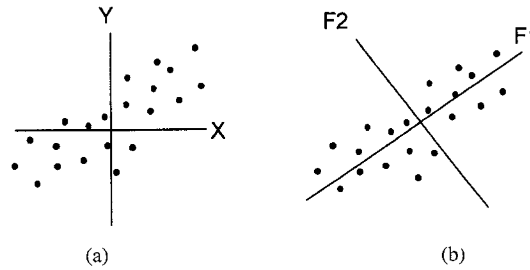

Fig. 2. Rotation of principal axis by the principal component technique on a correlated data set.

determined by the eigenvalues and original data. $\Lambda\mathbf{U}^{-1}$ is referred to as the factor pattern matrix because it contains the coefficients that will be multiplied by the principal components (factors) to reproduce the original data.

$$\mathbf{E} = \Lambda\mathbf{U}^{-1}\mathbf{F}. \tag{1}$$

One of the interesting aspects of the principal components technique is that the first eigenvalue, which corresponds to the first factor, is the largest since it is oriented in the direction responsible for most variation in the original data set. The second eigenvalue is the second largest because it is oriented, orthogonal to the first, in the direction responsible for the most of the remaining variation in the original data set. This continues until the nth eigenvalue explains the remaining variation. Using all n factors will describe all of the variation present in the original data. By using the inverse transform of (1) on each of the extracted FET parameters it is possible to derive a new data set which is completely orthogonal. Each of the new uncorrelated variables will be standardized according to (2) where $\bar{x}$ is the original data's mean and s_x is the sample standard deviation. In other words, the principal component variables have a mean of zero and standard deviation of one. By using the standardized uncorrelated data set in (1), the linear combination of the principal factors will produce the original data in standardized form. To restore the original FET parameters from a standardized data point x must be solved for in (2).

$$x_{\text{standardized}} = \frac{x - \bar{x}}{s_x}. \tag{2}$$

Most commercial statistical analysis packages will perform the principal component analysis on a data set. One such commercial statistical analysis package, SAS, will determine the new uncorrelated data set from an original data set, calculate the eigenvalues, cumulative variation explained by each of the new orthogonal factors, as well as the coefficients contained in the factor pattern matrix [12]. SAS can also be used to compute the means and standard deviations needed to restore the original FET parameters from the principal components.

Notice that no assumptions have been made of the original data's distributions. We may use the new orthogonal data set as it stands. However, if the original data follows a normal (Gaussian) distribution, the principal components will also have a Gaussian distribution because a linear combination of Gaussian distributions will be a Gaussian distribution. Each

TABLE I
COMMON MICROWAVE CIRCUIT STATISTICAL TESTS AND THEIR ASSOCIATED CUMULATIVE ERROR LEVELS

Experimentwise Significance Level	Pairwise α Level			
	Intrinsic FET Model		S-parameters (2-Port)	
	7 means or standard deviations	21 correlation coefficients	8 means or standard deviations	28 correlations coefficients
$\alpha_{\text{cumulative}} = 0.05$	0.0079	0.0024	0.0064	0.0018
$\alpha_{\text{cumulative}} = 0.075$	0.0111	0.0037	0.0097	0.0028
$\alpha_{\text{cumulative}} = 0.10$	0.0149	0.0050	0.0130	0.0038
$\alpha_{\text{cumulative}} = 0.15$	0.0229	0.0077	0.0201	0.0058

original FET parameter should be checked for this normality assumption by a statistical test. If the original data has a Gaussian distribution, or one can be obtained by data transformation, the derived principal components can be defined as having a standardized Gaussian distribution with a mean of zero and standard deviation of one. Equations can then be used to define the original data variables as a function of the principal components. This produces automatic interpolation of the original FET parameter database by simulating combinations of FET parameters that retain the correlations determined from the measured data but were never actually measured.

III. STATISTICAL EQUIVALENCE TESTING

Population equivalence testing needs to be done after performing the principal component analysis on the FET ECP database in order to confirm statistical model accuracy. In the past, there has been a serious lack of statistical rigor where modeling examples were shown to "agree well with" [13] or have "excellent" comparisons [9]. These comparisons of data are qualitative in nature and subjective at best. This section discusses the statistical tools that are available to determine a quantitative level of statistical accuracy when comparing measured and simulated microwave circuit populations.

There are two types of tests that can be applied to multivariate populations to determine some level of statistical equivalence. First, a multivariate distribution test can be used to determine if two populations are equivalent [14]. This test is the most accurate but unfortunately not commonly incorporated into many commercial statistical packages. The second method is using pair-wise comparisons of each variable's marginal density distribution. This does not provide sufficient conditions for multivariate statistical equivalence [15] except in the case of the multivariate Gaussian distribution. However, the pair-wise testing is useful even for non-Gaussian distributions because it can help identify which variables of two multivariate distributions are not statistically equal. Pair-wise testing of the distribution parameters can be easily achieved with the help of commercial statistical packages in the absence of a true multivariate distribution equivalence test. Most statistical texts cover pair-wise statistical testing [16]. The application of these tests to the more common microwave populations such as S-parameter data sets or ECP sets will be briefly described here.

Each statistical equivalency test is performed at a predetermined significance level (α) which is the probability of finding a difference between population statistics when there really is none. The person performing the test usually wants to keep this probability quite low, typically 0.05 to 0.1. Unfortunately, if a population has many different statistics to test, the probability of making an error accumulates according to (3) where "m" is the number of variables being pairwise tested

$$\alpha_{\text{cumulative}} = 1 - (1 - \alpha_{\text{pairwise}})^m. \tag{3}$$

For example, suppose a comparison of the means of two sample sets of S-parameters were to be made. There are four parameters, $S11, S12, S21$, and $S22$, each with a real and imaginary part. There will be eight means that would need to be compared to conclude statistical equivalence. In order to keep the cumulative error small of the entire statistical test, each pairwise α level must be very low. In fact, the cumulative error would be 0.57 if each test is performed at an $\alpha = 0.1$ level. That is, there would be a 57% chance of making an error if the S-parameter populations were found to be equivalent. If the pairwise comparisons were made at a $\alpha = 0.013$ level then the cumulative error would be 0.1 which is more acceptable.

Table I shows a compilation of suggested pairwise α levels and their corresponding cumulative α for different types of equivalence testing which are of special interest to microwave circuits. Significance levels for smaller or larger FET models or different size S-parameter networks may be derived in a similar fashion with (3). It can be seen that very low α-level pair-wise comparisons need to be made in order to keep the cumulative error low on any statistical tests.

IV. FET PARAMETER ORTHOGONALIZATION

This section illustrates the application of the Principal Component technique to statistical FET modeling. The methodology shown in Fig. 3 was applied to FET's produced in 1993 at the Texas Instruments GaAs Foundry in Dallas, TX. Each FET had four gate fingers and a total periphery of 300 μm. Fifty-four FET's were used from six 100 μm thick GaAs wafers with low-noise doping profiles. Normally, a sample size of only 54 FET's would be considered small for characterizing a FET population. However, the purpose of the study was to prove the usefulness of this statistical modeling methodology. Scattering and noise parameter measurements were obtained over the 0.5 to 26.5 GHz range at 0.5 GHz step intervals at

TABLE II
MEAN AND STANDARD DEVIATION OF EXTRACTED AND SIMULATED FET PARAMETERS

	Mean			Standard Deviation		
	Extracted	P.C.	$\pm\sigma$	Extracted	P.C.	$\pm\sigma$
Gm (mS)	92.335	92.152	92.195	5.251	5.160	5.153
Cgs (fF)	389.909	388.341	388.401	28.007	27.178	27.230
Ri (Ω)	2.594	2.600	2.598	0.312	0.314	0.302
Cds (fF)	79.178	79.219	79.217	2.762	2.696	2.757
Rds (Ω)	150.393	149.724	150.302	10.955	10.759	10.719
Cgd (fF)	32.207	32.213	32.147	3.169	3.148	3.202
Tau (ps)	2.520	2.513	2.524	0.224	0.220	0.221
Rg (Ω)	0.391	0.395	0.393	0.057	0.056	0.058
Rs (Ω)	2.539	2.538	2.547	0.190	0.181	0.191
Rd (Ω)	3.678	3.676	3.682	0.173	0.170	0.174
Vn	0.050	0.050	0.050	0.004	0.004	0.004
In	704.704	702.277	708.548	98.364	97.213	96.316
ReCorr	-3.088	-3.083	-3.093	0.167	0.163	0.172
ImCorr	-0.286	-0.290	-0.282	0.173	0.171	0.175
Rgs (Ω)	12388.0	12485.0	12348.2	2875.1	2821.5	2874.6

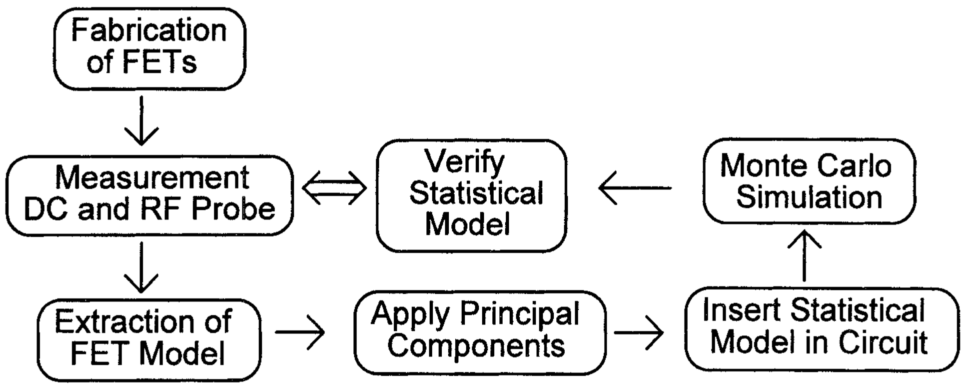

Fig. 3. Statistical modeling methodology flow chart.

the drain bias level of 3 V and 30 mA. Each FET's measured responses were used to extract the ECP values shown in Fig. 1. The ten intrinsic and extrinsic parameter values were obtained by analytical extraction of the FET parameters for each set of S-parameters similar to Anholt *et al.* [6] and Golio [17]. The five noise parameters, including R_{gs}, shown in the Fig. 1 were obtained by analytical extraction using the Hybrid-Pi noise model [3]. All ECP's were optimized to obtain a better fit to the individual FET measurements. Table II shows the mean and standard deviation values for all of the fifteen extracted ECP's.

All of the FET models were extracted and the commercial statistical analysis package SAS was used to determine the mean, standard deviation, and correlation matrix of the FET parameters. Table III shows the extracted database's correlation matrix with the statistically nonsignificant values shaded. Fifty-nine of the 105 correlation coefficients have a nonzero value when each was tested at an $\alpha = 0.05$ significance level. This strongly suggests the assumption of variable independence inherent to a Monte Carlo analysis would be violated. SAS was then used to determine the values in the factor pattern matrix shown in Table IV. The coefficients in the factor pattern matrix were multiplied by the principal component vector as shown in (1) to produce an equation for each of the FET ECP's. Each of the FET parameters derived from (1) can be placed in the equation block of a commercial CAD package such as Touchstone [8]. For example, the resulting expression for G_m is provided in (4) where $\bar{x}_{G_m}$ is the mean and sG_m is the standard deviation of the measured G_m sample. Of course, the expression for G_m can then be scaled to the desired FET periphery [4]

$$\begin{aligned} G_m = \bar{x}_{G_m} + S_{G_m} * (0.808 * F1 + 0.412 * F2 \\ - 0.006 * F3 - 0.050 * F4 + 0.323 * F5 \\ - 0.154 * F6 + 0.020 * F7 + 0.040 * F8 \\ + 0.149 * F9 + 0.028 * F10 - 0.134 * F11 \\ - 0.003 * F12 + 0.036 * F13 + 0.0216 * F14 \\ + 0.034 * F15). \end{aligned} \tag{4}$$

Fig. 4 shows the distribution of G_m values for the extracted database. The distribution seems to follow a Gaussian distribution although with just 54 samples the shape is not clearly defined. However, G_m along with all the other ECP variables each passed a statistical Shapiro-Wilk normality test at an

TABLE III
EXTRACTED FET PARAMETERS CORRELATION COEFFICIENTS WITH STATISTICALLY NONSIGNIFICANT VALUES SHADED

	Rgs	Im Corr	Re Corr	In	Vn	Rd	Rs	Rg	Tau	Cgd	Rds	Cds	Ri	Cgs	Gm
Gm	-0.46	0.26	-0.39	0.74	-0.67	-0.06	0.53	-0.72	0.05	0.54	0.12	0.17	-0.25	0.79	1.00
Cgs	-0.40	0.51	-0.14	0.41	-0.26	-0.08	0.32	-0.76	0.55	0.15	0.67	-0.16	-0.11	1.00	
Ri	-0.35	0.23	0.10	-0.48	0.51	-0.01	-0.01	0.41	0.58	-0.55	0.15	-0.06	1.00		
Cds	-0.01	-0.16	-0.38	0.28	-0.38	0.26	0.55	0.08	-0.49	0.19	-0.43	1.00			
Rds	-0.11	0.51	0.23	-0.14	0.40	0.00	-0.13	-0.44	0.78	-0.36	1.00				
Cgd	-0.15	-0.15	-0.65	0.90	-0.70	-0.12	0.29	-0.60	-0.59	1.00					
Tau	-0.31	0.53	0.28	-0.38	0.48	-0.05	-0.05	-0.09	1.00						
Rg	0.28	-0.36	0.48	-0.75	0.41	0.12	-0.29	1.00							
Rs	-0.33	0.13	-0.61	0.51	-0.52	0.43	1.00								
Rd	-0.06	0.00	-0.06	0.04	0.00	1.00									
Vn	0.08	0.27	0.60	-0.76	1.00										
In	-0.28	0.03	-0.69	1.00											
ReCorr	0.20	0.04	1.00												
ImCorr	-0.37	1.00													
Rgs	1.00														

TABLE IV
PRINCIPAL COMPONENT FACTOR PATTERN MATRIX EXPLAINING 96.6% OF TOTAL FET PARAMETER VARIATION

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7	Factor 8
Gm	0.808	0.412	-0.006	-0.050	0.323	-0.154	0.020	0.040
Cgs	0.046	0.822	-0.115	0.142	0.251	-0.009	-0.056	-0.025
Ri	-0.493	0.309	0.579	-0.463	-0.030	0.158	-0.129	-0.312
Cds	0.386	-0.425	0.559	0.110	0.266	0.202	0.363	-0.313
Rds	-0.183	0.850	-0.177	0.317	-0.054	0.185	-0.079	-0.227
Cgd	0.854	-0.232	-0.275	-0.181	-0.251	-0.071	0.015	-0.028
Tau	-0.383	0.862	0.144	-0.026	0.088	0.103	-0.219	0.068
Rg	-0.729	-0.513	0.323	-0.095	0.179	-0.109	-0.031	0.144
Rs	0.639	0.073	0.649	0.169	0.093	0.123	-0.073	0.174
Rd	0.50	-0.082	0.582	0.661	-0.289	-0.297	-0.153	0.004
Vn	-0.853	0.252	0.040	0.012	-0.296	0.023	0.168	-0.176
In	0.956	0.013	-0.095	-0.025	-0.166	-0.040	0.000	-0.044
ReCorr	-0.741	0.106	-0.245	0.182	0.314	-0.422	0.184	-0.033
ImCorr	0.015	0.735	0.138	0.002	-0.205	0.030	0.560	0.279
Rgs	-0.289	-0.488	-0.410	0.464	0.100	0.466	0.011	0.177

$\alpha = 0.05$ level of significance [18]. This indicates that the principal components will also follow a Gaussian distribution. Therefore, the variables $F1$ through $F15$ were defined in the Touchstone "variables" block to have a normal distribution with mean zero and standard deviation of one [8]. Notice when the statistical mode of the CAD package is not being used, that $F1$ through $F15$ will be at their nominal value, i.e., zero, and G_m will equal the mean of the entire FET sample. Also, the sum of the squares of the principal factor coefficients is equal to one which forces the standard deviation of G_m to be S_{G_m} during the Monte Carlo simulation.

The entire factor pattern matrix was used in (1) to implement all the FET parameters in terms of the principal component variables $F1$ through $F15$. One thousand samples were simulated using the Monte Carlo method on the principal factors and the simulated FET parameters were statistically analyzed. For comparison, 1000 samples were also simulated using only the FET parameter's mean and standard deviation which assumes independence of the FET parameters ($\pm\sigma$ method). Table II shows the means and standard deviations of the extracted FET parameters, the principal component model (P.C.) results, and $\pm\sigma$ model. Both the principal component model and the $\pm\sigma$ model are able to accurately reproduce the mean and standard deviations of the extracted FET parameters. In fact, both models produced means and standard deviations statistically equivalent to the original data with a cumulative $\alpha = 0.1$ level of error.

A representative example of correlation recovery for the principal component model, $\pm\sigma$ model, and the extracted data is shown in Fig. 5 for the correlation of C_{gs} with the

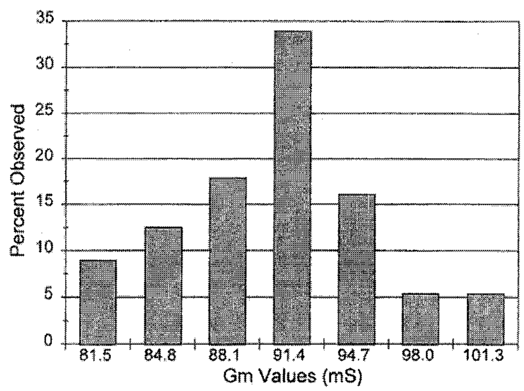

Fig. 4. Distribution of extracted G_m values.

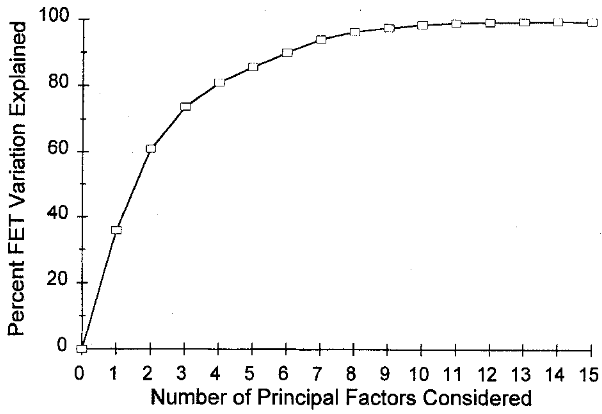

Fig. 6. Percent of FET variation explained as number of principal factors considered in model increases.

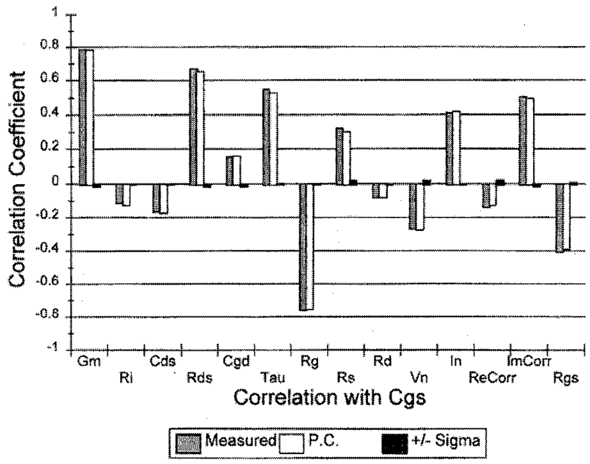

Fig. 5. Comparison of extracted and simulated C_{gs} correlation coefficient with other FET ECP's.

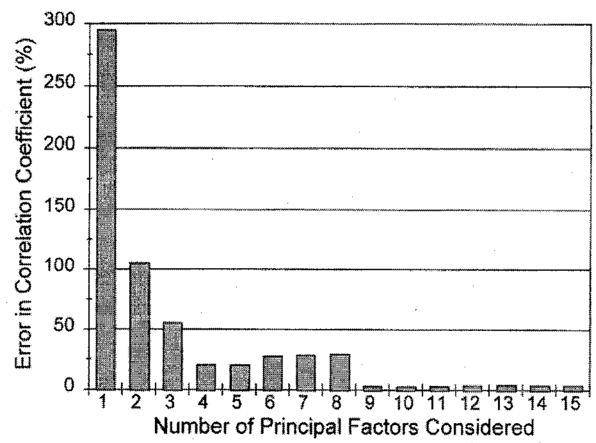

Fig. 7. Comparison of principal component model complexity correlation of G_m and R_i.

other FET ECP's. Fig. 5 demonstrates the principal component method recovered all of the measured C_{gs} correlations. Pairwise statistical tests verified that all 105 measured and principal component simulated correlations were statistically equivalent at a cumulative $\alpha = 0.1$ level of error. Fig. 5 illustrates the findings of these statistical tests including the fact that the $\pm\sigma$ method is not capable of modeling the FET parameter correlations because of the parameter independence assumption. Therefore, the $\pm\sigma$ method results in simulation of impossible FET parameter combinations during the Monte Carlo simulations. These examples show that principal components is a superior technique for accurate modeling of FET ECP statistical variations. Also, it was shown how easily and quickly the principal component model equations could be implemented into current commercial CAD products.

Fig. 6 shows that a larger portion of the total variation found in the original FET database is explained as the number of Principal Factors used in the model are increased. One hundred percent of the total variation is represented when the number of Principal Factors equals the number of original variables. Notice that the first nine factors explain 97.9% of the total cumulative variation in the FET parameters. This creates the potential to eliminate some of the less significant principal factors to produce a more compact model for each of the FET parameters [10]. To see how the number of principal component terms affect the model's correlation coefficients, the number of factors in the G_m principal component model was varied from all 15 to just the first principal component factor for a 1000 run Monte Carlo. Fig. 7 illustrates the large error possible for the simulated correlation coefficient between G_m and R_i when only a few principal factors are used. As the number of principal factors is increased the error decreases until it is statistically negligible. Notice that the error for this particular correlation coefficient is not strictly monotonic. The graph also shows the correlation can be adequately preserved with just nine factors instead of the original fifteen. Reducing the principal factors in the FET parameter statistical model can greatly decrease the complexity of the principal component model for the FET parameters. The number of terms in (4) could be decreased by approximately 36% using only factors $F1$ through $F9$. A smaller, less complex statistic model has many benefits such as easier implementation and faster simulation time.

The 15 ECP principal component equations were used in the FET model shown in Fig. 1 during a 500 run Monte Carlo simulation. The S-parameters from each run were stored in a S-parameter database and were used in comparison with the original measured S-parameter database to verify statistical

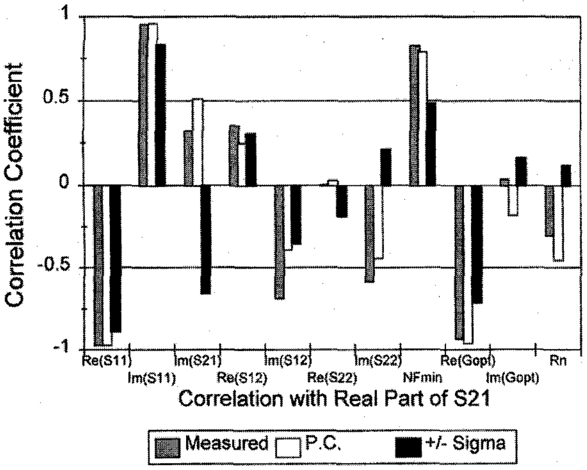

Fig. 8. Comparison of measured and simulated correlation coefficients: Correlation of real part of $S21$ with other FET Responses.

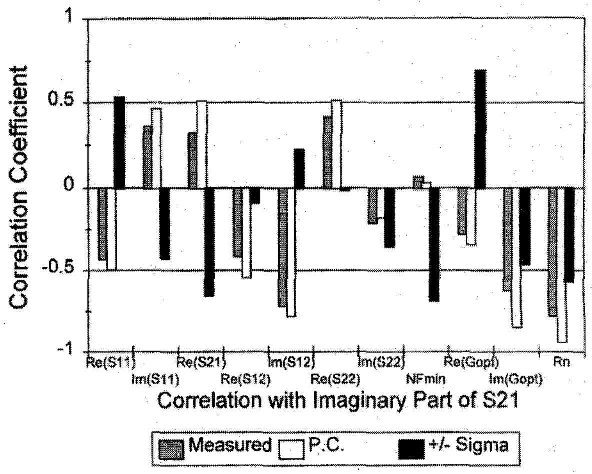

Fig. 9. Comparison of measured and simulated correlation coefficients: Correlation of imaginary part of $S21$ with other FET responses.

equivalence. Pairwise comparisons were made on the real and imaginary parts of the S-parameters and noise parameters which included $R_n, F_{\min}$, and the real and imaginary part of Γ_{opt}. All the measured and principal component simulated FET response means and standard deviations tested equivalent with an $\alpha = 0.05$ cumulative error. Fig. 8 depicts the correlation coefficient recovery for the measured and simulated FET real part of $S21$. Fig. 9 illustrates the same for the imaginary part of the FET's $S21$. The other FET reponses are similar to Figs. 8 and 9 in that the principal component model recovers the correlation coefficients fairly well. Equivalence tests between measured and simulated FET responses with a pairwise $\alpha = 0.05$ level of error showed that 54 of the 66 FET response correlations coefficients were statistically equal. Of those which failed equivalence, only two produced simulated correlation coefficients that were opposite in sign as the measured values.

A 500 run Monte Carlo simulation was also done using the traditional $\pm\sigma$ method for a comparison with the principal component method. The measured and simulated FET response means and standard deviations tested equivalent with a $\alpha = 0.05$ cumulative error level. Figs. 8 and 9 show the correlation coefficient recovery for the $\pm\sigma$ method as compared to the measured and principal component data. Both graphs illustrate that the $\pm\sigma$ model produces more significantly different correlation coefficients than does the principal component method. In fact, only 22 out of the 66 correlation coefficients tested equivalent at an $\alpha = 0.05$ level of error for the $\pm\sigma$ method of FET response simulation. Therefore, the $\pm\sigma$ model produced almost four times as many significantly different correlation coefficients than the principal component method. Of the significantly different correlation coefficients, 21 had a sign opposite to that obtained from the measured FET response database. This means the $\pm\sigma$ method was over three times more likely to produce an incorrect sign for those correlation coefficients that were significantly different. Both methods, $\pm\sigma$ and principal components, failed the cumulative pair-wise equivalence tests between measured and simulated FET response correlation coefficients and are therefore not statistically equal to the original measured database. However, it has been shown the principal component method is much more accurate at simulating the measured database than the $\pm\sigma$ model.

There are several reasons why the principal component model fell short of the goal to produce a statistically equivalent simulated database. First, there may be inadequate modeling of the individual FET's. This may have been caused by not including the extrinsic inductances in the FET model. The model optimization during ECP extraction may have also introduced inaccuracies during the ECP extraction due to local minima in the error functions. This extraction error could be indicated by the large percentage variances exhibited by R_g, R_{gs}, R_i, In, and Im_Corr all of which have been found to be difficult to extract. Anholt *et al.* make an excellent case suggesting that the quality of the statistical modeling of the FET is limited by the accuracy of the extraction method [6]. It is also possible that some of the ECP's exhibited a nonlinear relation which would cause errors when modeling the relation using a correlation coefficient. Correlation is defined as the linear relation between two variables and cannot accurately account for nonlinear relationships. This possibility was examined by producing scatter plots for all the FET ECP like shown in Fig. 10 for G_m and T_{au}. It might be inferred from Fig. 10 that there is a strong quadratic relationship present even though the correlation coefficient of 0.05 is quite low. Quantifying these nonlinear relations is beyond the scope of this paper but is a problem that will need to be overcome. Finally, nonnormal distributions for the FET ECP's may also be a definite problem because of the Gaussian assumption during the simulation of the Principal Factors. Normally, this problem could be diminished through the use of data transformations to get a more Gaussian distribution. However, the data presented in this paper covered two different lots of wafers which caused some of the FET ECP's to have distributions that could have been bi-modal. Larger number of FET samples would be needed to accurately test for this possibility. Bi-modal distributions may be caused by process shifts that will be hard, if not impossible, to model.

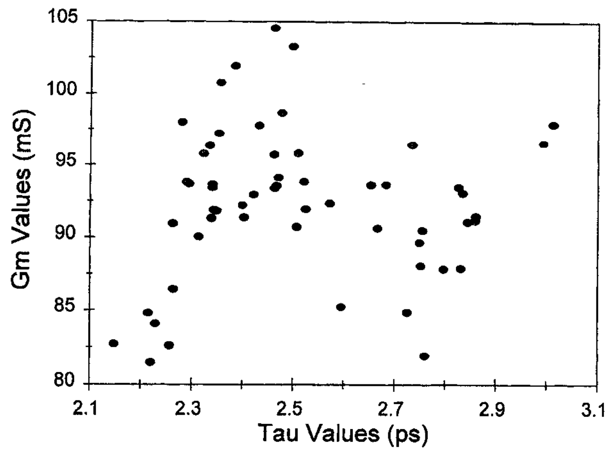

Fig. 10. Scatter plot of G_m and T_{au}: correlation coefficient = 0.05.

V. Other Applications

FET parameter orthogonalization has also been shown to pose a better conditioned model fitting problem [19]. Known correlations between the extracted FET parameters can be forced on a FET model optimization by using the principal component equations. Historical FET data or physics based models could be used for these known correlations. Principal components can also be applied to design of experiments (DoE) which requires orthogonal variables. The ability to reduce the model into a fewer number of principal factors than the original FET parameters will enhance the usefulness of FET variation modeling in DoE. Also, statistical population modeling could be used as a criteria to monitor the validity of active device parameter extraction. Once a population of FET's have been modeled, the methodology illustrated in Fig. 3 could be implemented to model the statistical FET electrical responses and compare them to the original database. Creating a statistically equivalent simulated population to a measured database is much harder modeling problem than representation of a single active device. Failure to successfully model the measured FET population could point out processing shifts, erroneous/nonphysical extraction, or an inadequate electrical model.

VI. Conclusion

Many of the prior works in statistical modeling of FET S-parameters are difficult to implement into current CAD software or are inaccurate in representing the FET population during Monte Carlo simulations. A new methodology for statistically modeling the extracted small signal FET parameters was developed and demonstrated. This method uses the principal component technique to orthogonalize the extracted FET parameters into a new set of variables called Principal Factors. Equations for the extracted FET parameters can then be written in terms of a linear combination of the orthogonal principal components and easily implemented into current commercial CAD software. The modeling approach was demonstrated on a small sample of 300 μm GaAs FET's and was statistically tested using techniques discussed in this paper. The principal component methodology was shown to preserve the extracted FET ECP's mean, standard deviation, and parameter correlations of a high level of statistical significance. Using the methodology significantly improved the ability to statistically model measured S-parameter and noise FET populations as compared to the assumption of ECP independence.

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